Att. 7
Ka and Broadband Conference
Isle of Ischia, Italy, Nov. 2003

Diversity Advantages for Nearby Sites, with Furuhama's Rain Correlation Function

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Abstract *We use Furuhama's rain correlation function to insert correlation coefficients into a bivariate exponential probability density function. We derive a new result for rain attenuation with switched site diversity. It indicates clear benefits for sites within 8 km, and the effective use of frequencies greater than 30 GHz. The bivariate exponential pdf also appears useful for cloud attenuation at much higher frequencies: A Soviet cloud autocorrelation function indicates effective diversity at 32 km for 75-98 GHz systems.*

1. Background

Rain attenuation has often discouraged satellite communication system designers from designing systems for the millimeter wave region. Rain attenuation can be severe for frequencies greater than 30 GHz, and designers have tried various methods to alleviate the rain attenuation. One key method has been the use of ground site diversity. Some diversity analysis indicates that ground site separation must be much greater than 10 km to achieve significant advantages for diversity. We develop analysis here to indicate that outstanding advantages can be found for distances often less than 10 km, and sometimes less than 5 km. We start with Furuhama's autocorrelation function for rainfall, then use a bivariate exponential probability density function (pdf) to derive a general attenuation exceedance probability for separated sites. The equation is then reverted to find a new equation for attenuation as a function of probability to meet the needs of communication engineers. Attractive frequencies well into the millimeter wave region will be indicated by the new results.

Some of the valuable early insights into the benefits of diversity centered on the concept of 'Diversity Gain.' Prof. D. Hodge of Ohio State University developed explicit results (1) for Ku band systems, with helpful equations as:

$$G=\left\{1-e^{-0.46d\left\{1-e^{-0.26A}\right\}}\right\}\left(A-3.6\left(1-e^{-0.24A}\right)\right) \quad \text{dB}$$

where d= site separation (km), A=single site attenuation (dB) (1-1)

$$a=A-3.6\left(1-e^{-0.24A}\right) \quad \text{dB, and}$$

$$b=0.46\left(1-e^{-0.26A}\right) \quad \text{Km}^{\wedge}(-1)$$

Clear advantages were shown for sites separated by over 10 km, and O.S.U went on to show advantages at the 30/20 GHz band.

Later, Morita and Higuti (2) recognized that it would be helpful to analyze the fundamental properties rain attenuation in order to generalize the diversity advantages to other distinct cases.

They used the powerful Lin (3) lognormal rain attenuation model to evaluate joint exceedance probability from two correlated sites. The transcendental result was so long that it offered intractable difficulty in reverting it for a general attenuation at two sites. A linear regression allowed helpful but limited insights into other frequencies and elevation angles.

We seek general results for a wide range of frequencies and elevation, and fundamental properties of rain cell sizes are required. Fortunately, Furuhama and Ihara (4) recognized that systematic and large scale efforts were needed to describe the effects of rain cell sizes.

2. Analysis

Furuhama and Ihara developed correlation functions to describe the rain rate relations between two separated ground stations. The correlation functions were seasonal, with large scale characteristics for hurricanes and much smaller sizes for most rains. Fig. 2-1 shows correlation function results for remarkably different seasons, as for a Typhoon season and for the month of July. Fig. 2-2 includes the results for all the rain.

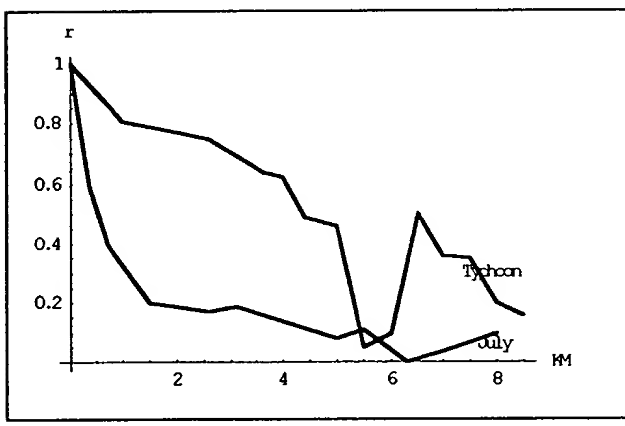

Fig. 2-1 Rain Correlation Function for Typhoon (top), July

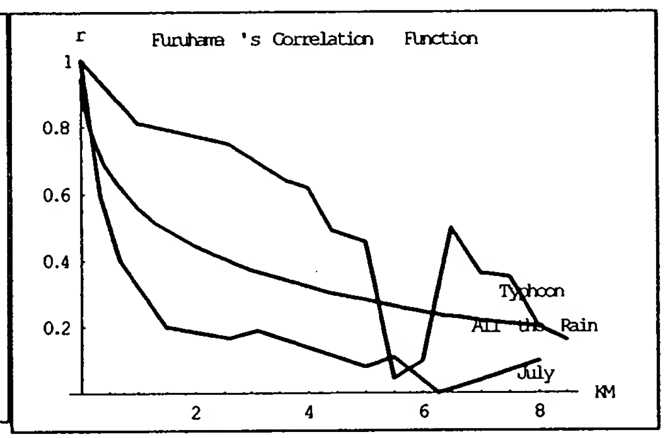

Fig. 2-2 Correlation Function for All Rain

Furuhama recognized that a correlation for all the rain (middle curve, Fig. 2-2) could be represented conveniently by

$$\boxed{R(x) = e^{-0.57\sqrt{x}}} \qquad (2\text{-}1)$$

where x= separation, km

Furuhama and Ihara's insight into the importance of the correlation function will next be seen with a description of Davies' bivariate exponential probability density function.

Furuhama's function can be used as a direct input into a correlated bivariate exponential pdf, and then to develop quantitative results for diversity advantages. We use the form of the correlated exponential pdf as:

$$p(A1,A2) = \frac{e^{-\frac{A1+A2}{B(1-r)}}\, p\, I_0\left(2\sqrt{\frac{A1\,A2\,r}{B^2(1-r)^2}}\right)}{B^2\,(1-r)}$$

(2-2)

where A1 and A2 are the rain attenuation (dB) at the 2 sites
r= correlation between sites, found from Furuhama
B=standard deviation of the univariate pdf, and
p= conditional probability that rain attenuation is observed; as .01 or .02

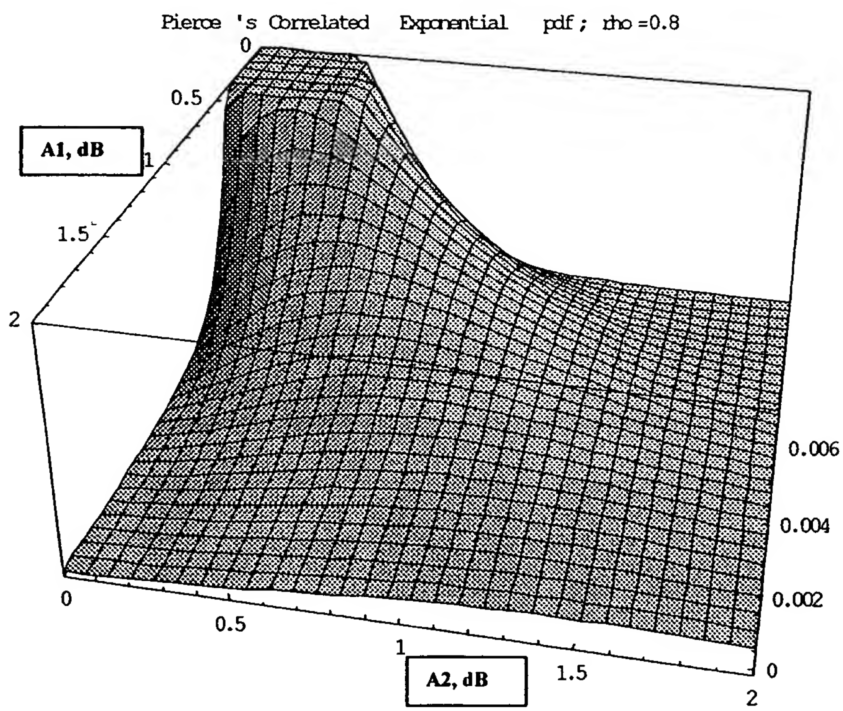

Fig. 2-1 Pierce's Correlated Exponential pdf (Eq. 2-2)

The double integral on the density function should be evaluated to find the joint probability of exceeding arbitrary attenuation levels. The probability of both sites having attenuation greater than AR (dB) may be functionally shown as Eq. 2-3.

P[A1>AR, A2>AR]=

$$\frac{p \int_{AR}^{\infty} \int_{AR}^{\infty} e^{-\frac{A1+A2}{B(1-r)}} I_0\left(2 \sqrt{\frac{A1\,A2\,r}{B^2 (1-r)^2}}\right) dA2\, dA1}{B^2 (1-r)} \tag{2-3}$$

The low values of the Furuhama correlation function (r) will be key to finding low probability of attenuation AR. System operators will recognize AR (dB) as the rain attenuation available with switched diversity, when they can choose the site with the least rain attenuation.

The integral has not yielded to attempts to integrate it exactly, and it turns out to be a very lengthy numerical integration. An upper limit is chosen as (10 B) rather than infinity. The exceedance probability (Eq. 2-3) is abbreviated as (pr) below.

The result of the double integration can be found as Figures 2-2 and 2-3. Fig. 2-2 shows exceedance probability (PR=Log10(pr)) v. normalized (AR/B) and correlation coefficient r. Exceedance probability contours may also be shown as Fig. 2-3.

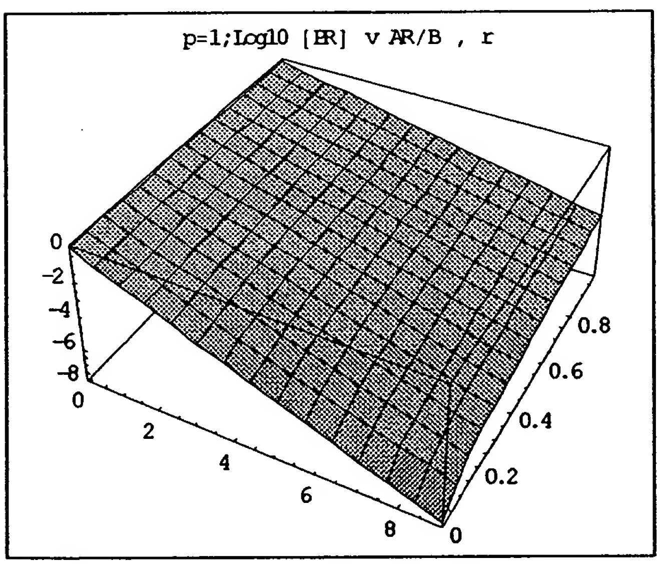

Fig. 2-2 PR v. (AR/B), r

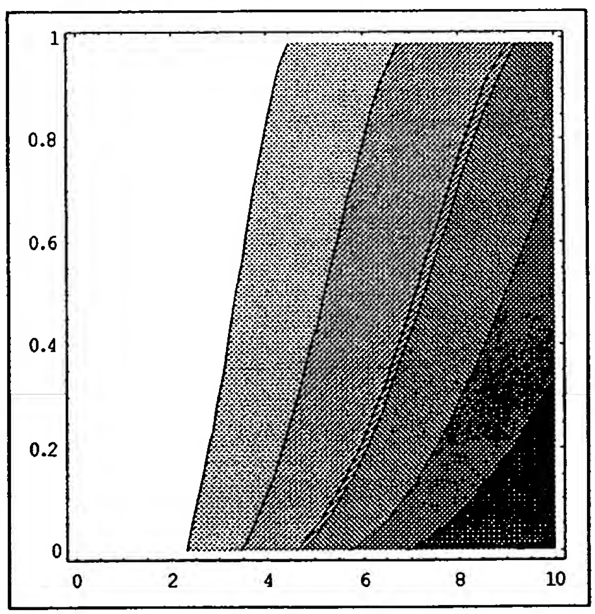

Fig. 2-3 PR Contours v. (AR/B), r

The bivariate function of Fig. 2-2 is defined for all possible weather events, even clear weather at both ground sites. Instead, the exponential density function should be defined for rain events in the 1% to 0.1% range: This is where the exponential pdf has the most relevance.

The communications engineer has a large problem remaining, even after the bivariate pdf has been solved. Eq. 2-3 expressed probability as a function of attenuation AR and standard deviation of the exponential function. This equation should be reverted for AR, as it was for the special condition of low correlation (r<0.2) and large separation distance (d> 8km) in 1983 (5). Eq. 2-3 can be reverted, with a close approximation, to give more general results suitable for nearby sites.

$$(AR/B)= 125.066\sqrt{(r-1.49042)(r-1.28261)(r^2-0.576796r+0.563242)(r^2-0.412372r+0.726974)} - 127.112\sqrt{0.0157342\,PR+0.968061(r-1.45741)(r-1.35762)(r^2-0.535312r+0.699923)(r^2-0.431856r+0.596869)}$$

dB (relative to pr=.01) (2-4)

where PR=Log10[Exceedance probability]

This new result can be plotted as Fig. 2-4. Fig. 2-4 assumes that interesting rain attenuation occurs only 1% of total time, so PR begins at Log10[.01]= -2. All rain attenuation inferred from Fig. 2-4 should then have the low 1% attenuation added to get the final estimate.

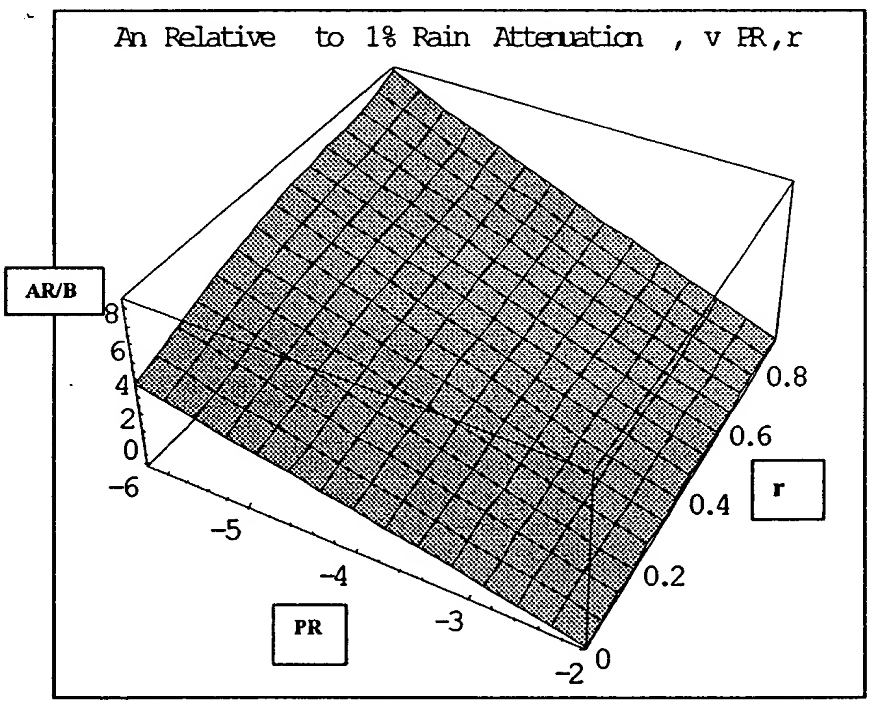

Fig. 2-4 Normalized (AR/B) v. Log10[Exceedance Prob], Correlation r

This result for switched diversity is related to attenuation in satellite communication systems in the next section.

3. Applications to Satellite Attenuation

Exponential probability density functions are often observed for rain attenuation on satellite links, for probabilities ranging between 0.01 and 0.001. This range, for light to moderate rain attenuation, will be of primary interest for us because switched diversity will be a powerful weapon against higher attenuation. We extend a Crane rain model (6) to include sharply rising attenuation with frequency with the aid of G.T. Wrixon's Sun Tracker studies (7). The Sun Tracker studies showed attenuation (dB) tended to increase as $f^{1.81}$ for the 16 to 90 GHz range. Fig. 3-1 shows single site attenuation at New

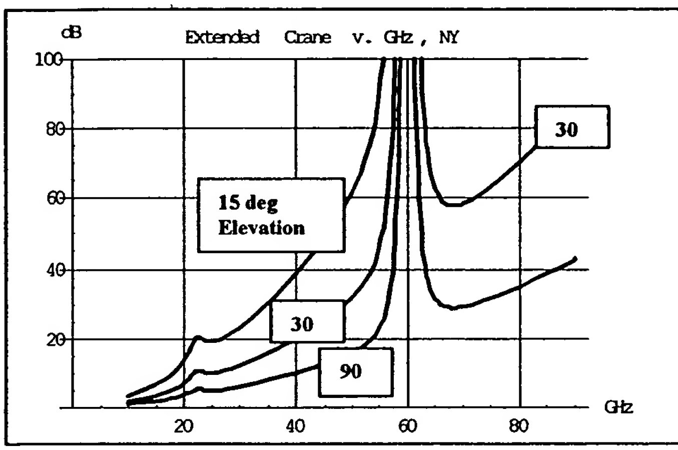

Fig. 3-1 Attenuation at NY pr=.001
pr=.001 (PR= -3) 12 mm/hr

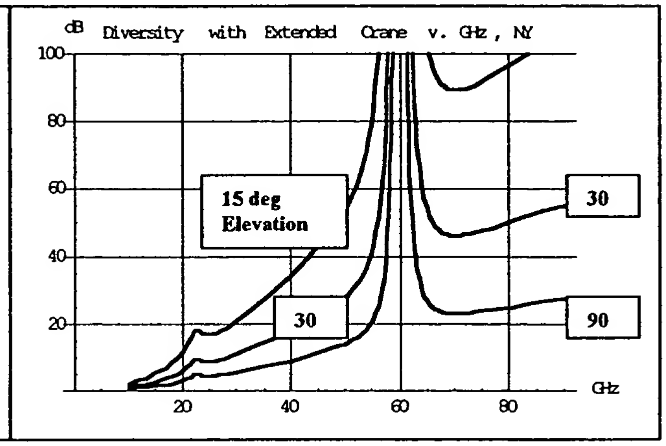

Fig. 3-2 Switched Diversity at NY
8km Site Separation mot-NYb

York. Fig. 3-2 indicates the attenuation advantages of 8 km site diversity, especially for frequencies greater than 70 GHz. Fig. 3-3 shows net loss (loss – gain) for constant aperture antennas, using the benefits of 8 km site separation.

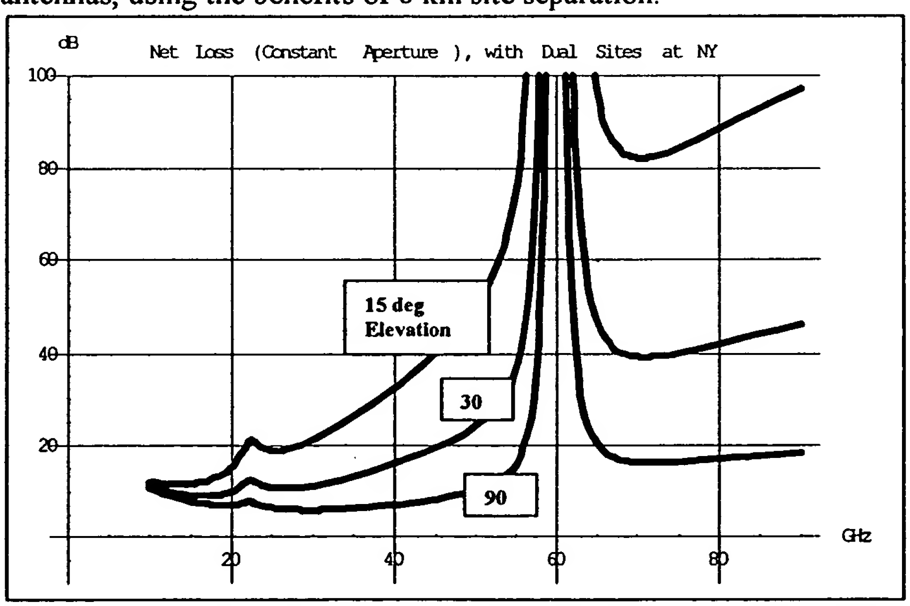

Fig. 3-3 Net Loss v. Frequency at NY, with Site Separation= 8km

Fig. 3-3 indicates that 30-40 GHz links would be expected to do well at 99.9% availability for satellite passes near zenith. The higher satellite bands, as 70-80 GHz, would not be expected to do as well at this moderately rigorous availability. Lower availability, or wider site separation, or 8 dB penalty, would be indicated at 70- 80 GHz.

75-98 GHz Satellite Links

Barbaliscia (8) cogently observed that many satellite systems are quite worthwhile with only 95- 99% availability. These systems with modest availability might be able to simply ignore rain for systems planning: However, they would still need to pay close attention to cloud cover. Fortunately, Soviet space studies (9) paid close attention to cloud autocorrelation functions. Boldyrev and Tulupov derived interesting properties of cloud cover, deriving a function as:

$$\text{Boldyrev } R(x)=0.2\,e^{-x}-e^{-x\gamma 1}+e^{-x\gamma 2}+0.8\,e^{-x\alpha}\,\mathrm{Cos}[x\,\beta] \qquad (3\text{-}1)$$

with

$\alpha=0.003 \quad \beta=0.0075 \quad \gamma 1=0.015 \quad \gamma 2=0.036$

Eq. 3-1 may be shown as Fig. 3-4.

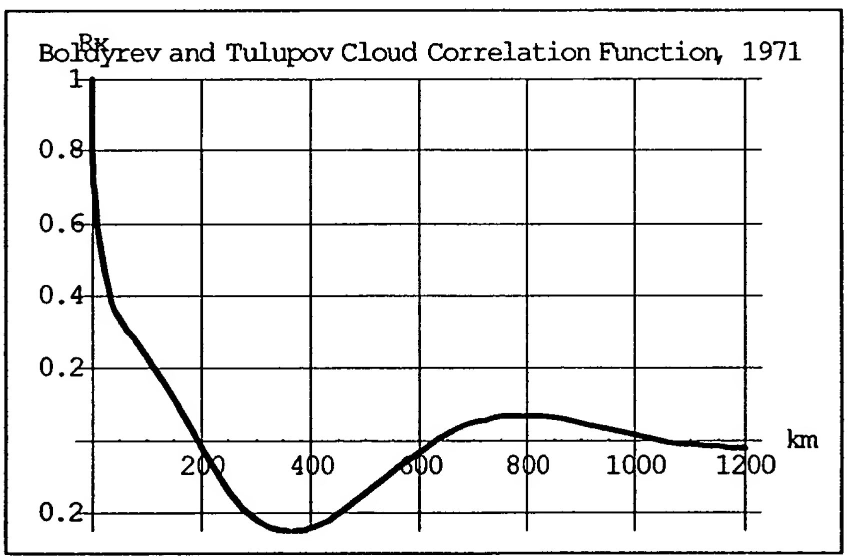

Fig. 3-4 Soviet Cloud Autocorrelation Function v. Distance (km)

The function is interesting in several ways, including the drop to *negative* values at 200 km. This would lead to a separate discussion about surprisingly robust diversity studies for rain attenuation which occurred at 200-300 km. We cannot go into that here, however, and we direct our attention to the correlation at distances less than 40 km. A detailed look at Fig. 3-4 would reveal correlation as 0.4 at 32 km. A millimeter wave satellite system with single link availability as 90% (10) would be expected to improve its availability to almost 97% with two sites separated by 32 km. Two sites separated by 200 km should expect 99% non rainy availability.

This kind of diversity is less expensive than it appears: Separate NASA sites at White Sands, New Mexico normally serve as separate data links, but could be used to serve a single priority link *in extremus*.

Conclusions

Furuhama's rain correlation function has received relatively little attention, perhaps because the relation to communication link availability was not obvious. We applied the correlation function to a bivariate exponential pdf, and we developed a new result for net attenuation for switched diversity, as shown by Eq. 2-4. Relatively nearby sites are indicated to allow frequencies much higher than 30 GHz to be considered for high elevation satellite systems of reasonably stringent availability (0.999). The lower availability requirements for VSAT systems would benefit from Boldyrev's cloud autocorrelation function. Frequencies in the 75-98 GHz region could be strongly considered for VSAT systems in large parts of the temperate region, including New York and Rome.

Acknowledgements

Dr. W. W. Wu, Founder of Advanced Technology Methods (ATMCO), recognized the significance of Furuhama's rain correlation functions and offered valuable discussions. Also, the radar observations of Dr. Julius Goldhirsh tend to group the maximum sizes of intense rain cells in the 3-5 km range. These direct observations might be extended to a more general correlation function, with higher intensities at smaller scale sizes.

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